An Algebraic Attack on the Bluetooth Key Stream Generator

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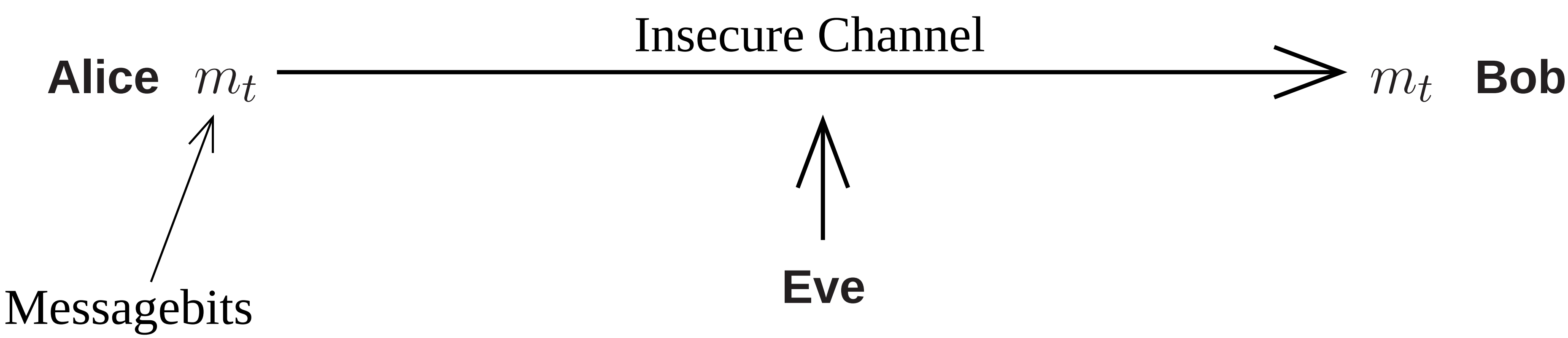
Bluetooth is a standard for wireless communication between different devices (e.g. mobile phones, desktop computers, . . .), making pervasive computing possible.

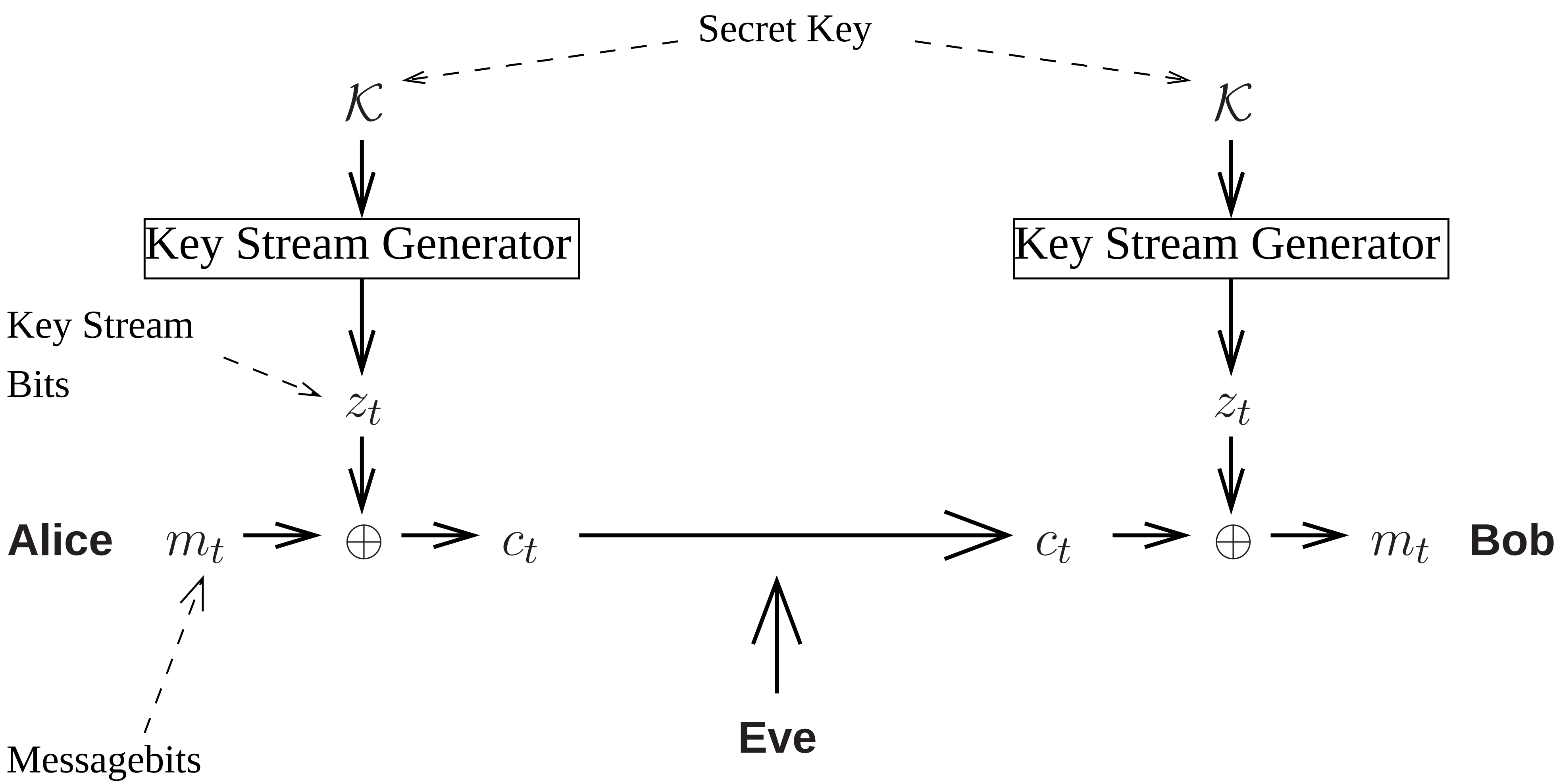
How secure is the Bluetooth encryption?

Situation

Alice wants to send a message $m = (m_1, m_2, \ldots)$ to Bob without being eavesdropped.



Using Key Stream Generators



The Eavesdropper Eve

It is assumed that Eve knows:

- The key stream generator
- Some of the key stream bits z_t

Eve knows not:

- The secret key $\mathcal{K}$

<u>Eve tries to recover the secret key $\mathcal{K}$.</u>

Algebraic Attacks

Algebraic attacks came up in the last years. There exist algebraic attacks against

- Block ciphers
  - AES, Serpent (Courtois, Pieprzyk; 2002)
- Stream ciphers
  - Toyocrypt, LILI-128 (Courtois, Meier; 2003)
  - Bluetooth key stream generator (Armknecht; 2002)

Algebraic Attacks

Simply spoken, an algebraic attack consists of two steps:

1. Set up a system of equations, the unknowns being the bits of the secret key $\mathcal{K}$.
2. Solve it.

Key Stream Generator I

Linear feedback shift register (LFSR) of length n:

$$\boxed{\text{LFSR}} \longrightarrow z_t$$

$$\mathcal{K} = (x_1, \ldots, x_n)$$

Advantages:

- Very fast
- z_t seem to be randomly chosen

Disadvantage:

- For each clock t, there exists a known linear function F_t with $z_t = F_t(x_1, \ldots, x_n)$

Algebraic Attack on LFSRs

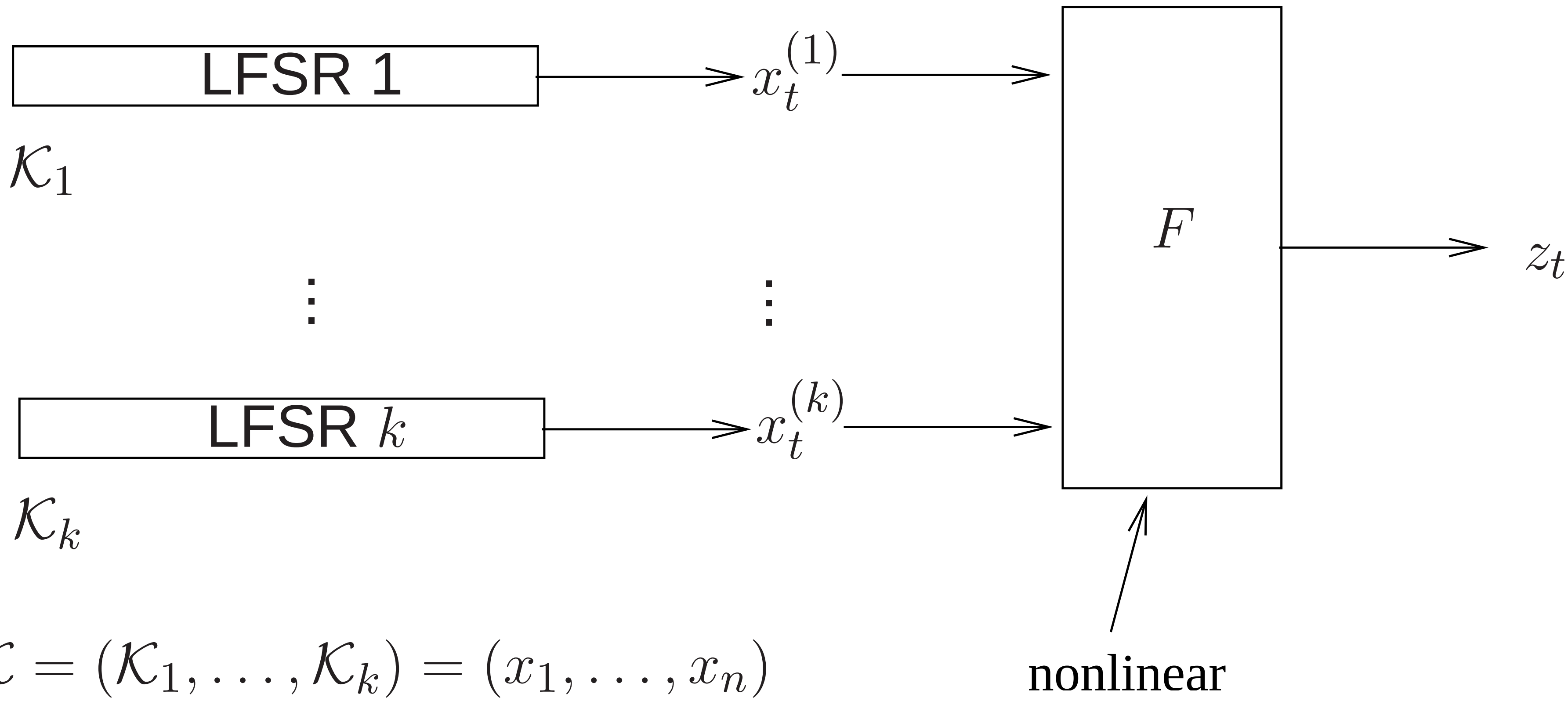
Algebraic attack on LFSRs:

1. Set up system of linear equations

$$\begin{aligned} z_1 &= F_1(x_1, \ldots, x_n) \\ z_2 &= F_2(x_1, \ldots, x_n) \\ &\vdots \end{aligned}$$

2. Solve this system of equations (easy, as it is linear)

A combiner with k LFSRs:



$$\mathcal{K} = (\mathcal{K}_1, \ldots, \mathcal{K}_k) = (x_1, \ldots, x_n)$$

Algebraic Attack on k-combiners

Algebraic attack on k-combiners:

1. Set up system of <u>nonlinear</u> equations

$$\begin{aligned} z_1 &= F(x_1^{(1)}, \ldots, x_1^{(k)}) = F_1(x_1, \ldots, x_n) \\ z_2 &= F(x_2^{(1)}, \ldots, x_2^{(k)}) = F_2(x_1, \ldots, x_n) \\ &\vdots \end{aligned}$$

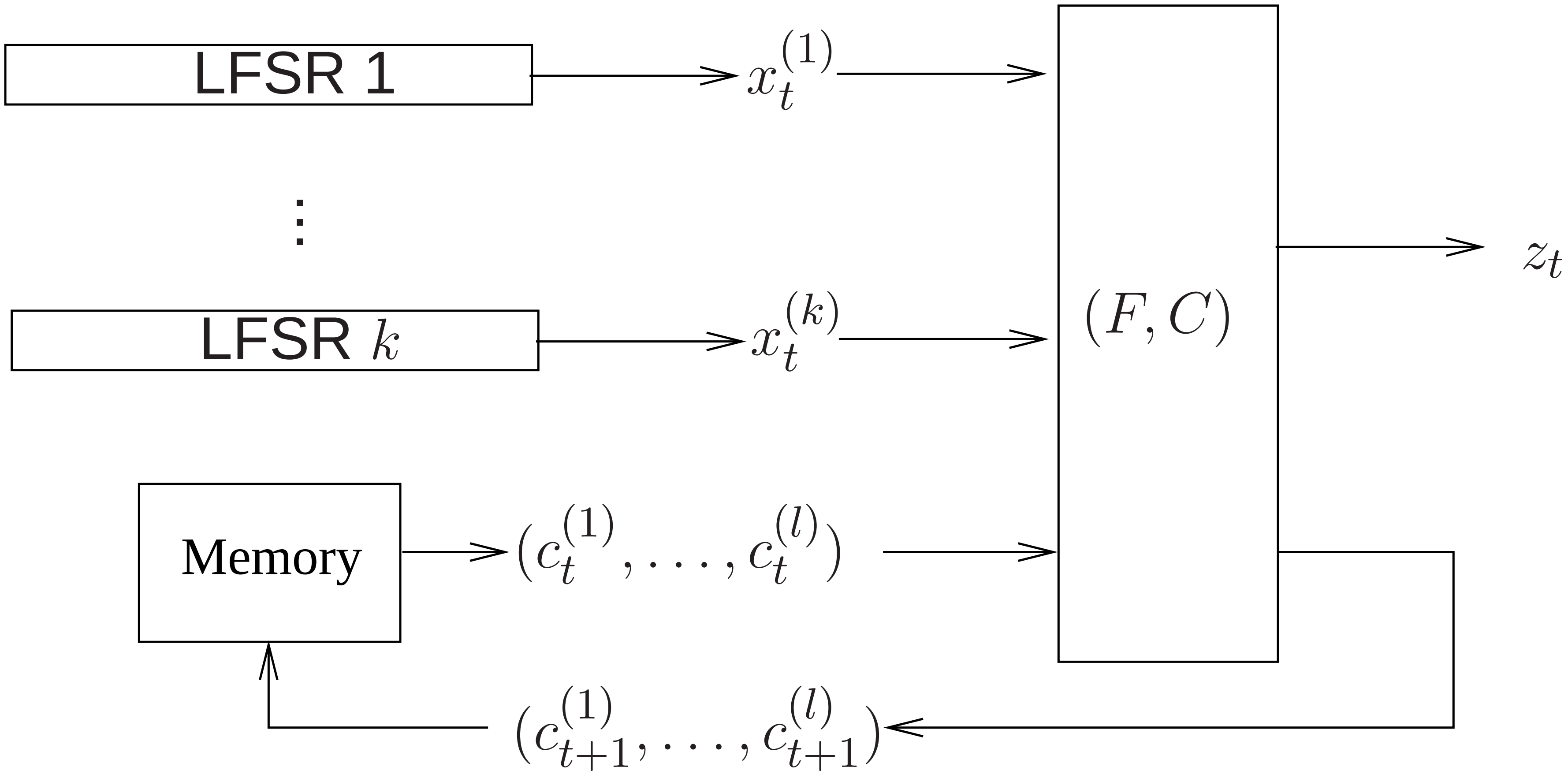
   Each F_t has the same degree d.

2. Replace each monomial of degree > 1 by a new variable (Linearization) $\Rightarrow$ a system of linear equations in $\approx \binom{n}{d}$ unknowns.

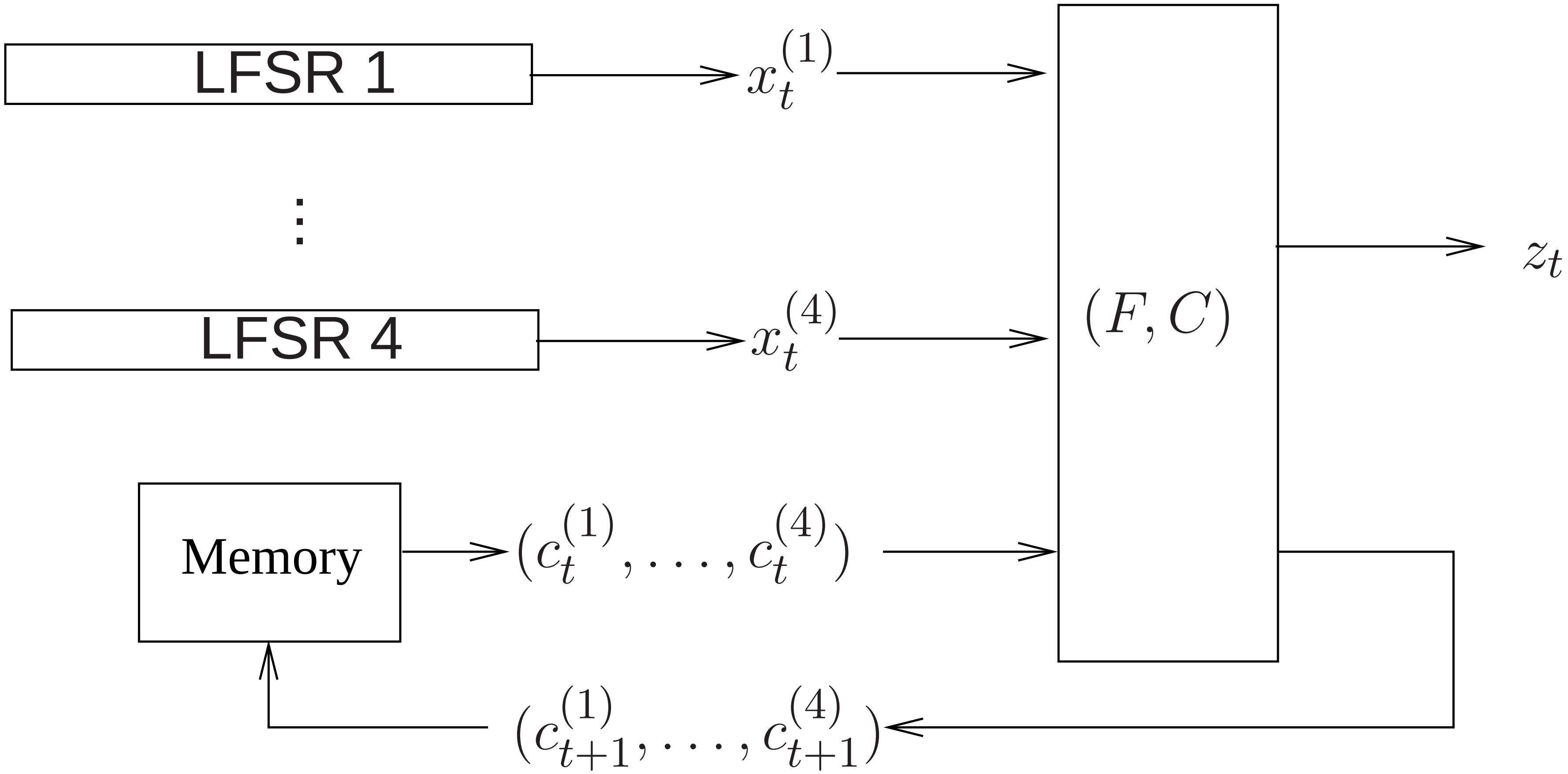
3. Solve this system of linear equations.

Key Stream Generator III

A combiner with k LFSRs and l memory bits:



The Bluetooth key stream generator:



Algebraic Attack on Bluetooth?

1. System of equations:

$$\begin{aligned}
& \vdots \\
z_t &= F(x_t^{(1)}, \dots, x_t^{(4)},\ c_t^{(1)}, \dots, c_t^{(4)}) \\
&= F(x_t^{(1)}, \dots, x_t^{(4)},\ C_t(x_1^{(1)}, \dots, x_{t-1}^{(4)}, c_1^{(1)}, \dots, c_1^{(4)})) \\
&= F_t(x_1, \dots, x_n, c_1^{(1)}, \dots, c_1^{(4)})) \\
& \vdots
\end{aligned}$$

   In general, the functions F_t have a high degree.

2. Solving ???

A relation without memorybits

Surprisingly, there exists a relation $\tilde{F}$ of degree 4 with

$$0 \;=\; \tilde{F}(X_t, X_{t+1}, X_{t+2}, X_{t+3},\; z_t, z_{t+1}, z_{t+2}, z_{t+3})$$

where

- $X_t = (x_t^{(1)}, x_t^{(2)}, x_t^{(3)}, x_t^{(4)})$ is the output of the 4 LFSRs at clock t
- $z_t, z_{t+1}, z_{t+2}, z_{t+3}$ are four sucessive bits of the <u>known</u> keystream

<u>This relation depends NOT on the memory bits!</u>

Algebraic Attack on Bluetooth!

1. Set up the following system of equations

$$
\begin{array}{rcl}
 & \vdots & \\
0 & = & \tilde{F}(X_t, X_{t+1}, X_{t+2}, X_{t+3},\ z_t, z_{t+1}, z_{t+2}, z_{t+3}) \\
 & = & \tilde{F}_t(x_1, \ldots, x_n,\ z_t, z_{t+1}, z_{t+2}, z_{t+3}) \\
 & \vdots &
\end{array}
$$

2. Linearization $\Rightarrow$ system of linear equations with $\approx 2^{23.07}$ unknowns.
3. Solve it. Work effort $\approx 2^{67.58}$ operations.

Alg. attacks on comb. with memory

Theorem (Krause, Armknecht; 2003)

For each combiner C with k LFSRs and l memory bits, a nontrivial relation $\tilde{F}_C$ of degree $\lceil k(l+1)/2 \rceil$ with

$$0 = \tilde{F}_C(X_t, \ldots, X_{t+l}, z_t, \ldots, z_{t+l})$$

can be constructed.

$\Rightarrow$ Algebraic attacks are always possible on combiners with memory!